Evaluation of different methods for spatial analysis of Cobalt fractions in the Indo-Gangetic Alluvial Plains

M.S. Grewal, R.S. Raman and M.S. Kuhad

Department of Soil Science, CCS Haryana Agricultural University, Hisar 125 004, India

Abstract

Surface samples from 72 locations at a grid of 10 km x 10 km, from a part of the Indo-Gangetic alluvial plain of Rohtak district were collected and analysed for exchangeable cobalt (Exch-Co), and carbonate cobalt (Carb-Co) having CV values 37 and 61 per cent, respectively. Both the forms of cobalt were normally distributed. The semivariograms were unbounded and expressed by power function regression models with r=0.893 (Exch-Co) and r=0.807 (Carb-Co). The isorithmic (contour) maps prepared by block kriging were smoother than the point kriged maps. The estimation variance and sample size by block kriging was lower than classical and point kriging models.

Additional keywords: Spatial variability, cobalt fractions,

Introduction

Spatial variation in soil is a practical problem affecting the reliability of soil testing for better management practices. Numerous studies have been conducted to examine the various aspects of this problem and reviewed in details by Beckett and Webster (1971), Dahiya et al. (1984), Trangmar et al. (1985) and Webster (1985).

Spatial variability of exchangeable and carbonate cobalt has so far been evaluated by classical statistical methods, which consider the data as spatially independent (random distribution). The geostatistical evaluation has proved that many soil properties are not randomly distributed but they show distinct spatial correlation structure (Goldin and Lavkulich 1988; Campbell et al. 1989; West et al. 1989). This new approach affords a mean of quantifying the spatial dependency among sampling points and also allows unbiased estimates of interpolated values. The study, therefore, was undertaken with the objectives to analyse the extent of variation, determine the nature of spatial dependence of Exch-Co and Carb-Co, estimate, display and interpret semivariograms as a measure of continuity and design optimum sampling strategy.

Materials and methods

The study area comprising of 5600 km² of the Rohtak and adjoining district of Haryana, it lies between 28°30' to 29°05' 30"N latitudes and 76°12'30" to 76°58' E longitudes (Fig. 1). The landscape is a part of the Indo-Gangetic alluvial plains. It was formed by alluvial deposition brought by Himalayan rivers. The alluvial deposits consist of sand, silt and clays with occasional gravel beds (Dahiya et al. 1988) with the exception of few small out liers of Alwar quartzite. There is no sign of hard rock exposures in the area and is almost concealed by the wide expanse of alluvium. The climate is sub-tropical, semi-arid, continental and monsoonal type. The total annual rainfall is 535 mm. The soils have light to medium texture with alkaline reaction (pH 8.0) and the alluvium is generally calcareous. Illite and smectite are the dominant clay minerals.
Seventy two surface soil samples at 10 km x 10 km grid were collected. From each location five samples were taken randomly, mixed together to get a composite sample. These were analysed for exchangeable cobalt (Exch-Co) and carbonate cobalt (Carb-Co) by Tessier et al. (1979) method.

Statistical approach

Statistical analysis was performed to examine the hypothesis that the data were normally or log-normally distributed using the frequency distribution function by con-
structing cumulative probability plots on normal probability paper. The straight lines of such plots indicated normal frequency distribution of these nutrients. The number of samples required to estimate the true mean was calculated by using the equation reported by Cline (1944):

\[
N = \left[ \frac{(t.s)}{D} \right]^2 \tag{1}
\]

where, 'N' is the estimated sample size required to be within 'D' units of the mean, 't' is the Student's t-value for the desired confidence level of the estimate and 's' is the standard deviation.

The theory of regionalized variables (Matheron 1971) was used to investigate the spatial variability of this nutrient. The semivariance function (h) is expected to have the expected square difference between values at locations separated by a given lag and is used to express spatial variations (Journel and Huijbregts 1978). On the transect, where 'n' observations were taken at a regular interval \( z(i) \), where \( i = 1, 2, \ldots, n \), semivariances can be calculated using the equation:

\[
G(h) = \frac{1}{2N(h)} \sum [z(x_i) - z(x_i + h)]^2 \quad i = 1, \ldots, n \tag{2}
\]

where \( G(h) \) is the sample semivariance and \( N(h) \) is the number of pairs of data points separated by the distance \( h \). \( z(x_i) \) and \( z(x_i + h) \) are the values of the property at locations \( x \) and \( x + h \) separated by the vector \( h \). This is known as the lag. The jackknifing (\( r^2 \)) procedure was used to test the adequacy of selected semivariance model. The semivariograms were well described by power regression function model.

Kriging was performed using the procedures described by Journel and Huijbregts (1978) and Burgess and Webster (1980). Punctual kriging, which is an exact interpolator (Delhomme, 1978) represented by equation (3), was used to estimate values of cobalt for unsampled locations.

\[
z(x_o) = \sum \lambda_i z(x_i) \quad i = 1, \ldots, n \tag{3}
\]

where each estimated value \( [z(x_o)] \) is a weighted average of observed values \( [z(x_i)] \) within the neighbourhood of kriging location, and \( \lambda \) are the weights on each sampling location.

Users of information on soil are often interested in average estimates of discrete areas or blocks (e.g. management units) rather than point estimates. Using the appropriate semivariograms both the forms of cobalt were interpolated at 5-km intervals both by point and block kriging. The kriged value \( z^* \) for any block \( v \) is a weighted average of the observed values \( x_i \) in the neighbourhood of block i.e.

\[
z^*(v) = \sum \lambda_i z(x_i) \quad i = 1, \ldots, n \tag{4}
\]

The estimation variance, \( s_{kb}^2 \), is given by

\[
s_{kb}^2 = \sum \lambda_i G(x_i, v) + uv - G(v, v) \quad i = 1, \ldots, n \tag{5}
\]
where \( G(x_i, v) \) is the average semivariance between the sample points \( x_i \) in the neighbourhood and those in block \( v \). \( G(v, v) \) is the average semivariance between all points within \( v \) (i.e. within block variance) and \( u \) is the lag range parameter.

**Results and discussion**

The coefficient of variation (CV) was used to express the variability of these forms of cobalt. The CV value was 37 per cent for Exch-Co and 61 per cent for Carb-Co. Both these forms have medium variations. The Exch-Co ranged from 0.12 to 1.2 and Carb-Co from 0.012 to 0.33 mg/kg.

The semivariograms of Exch-Co and Carb-Co were unbounded showing no definite sill and range. Both the forms were well described by power regression function model with \( r = 0.893 \) (Exch-Co) and \( r = 0.807 \) (Carb-Co). The nugget variances were 0.004 for Exch-Co and 0.000397 (mg/kg)^2 for Carb-Co i.e. 8 and 7 per cent, respectively for the sample variance (Fig. 2).
The isorithmic maps of Exch-Co and Carb-Co prepared by point kriging were very spotty and this spottiness disappeared to a great extent in case of block kriged maps. There is a general tendency of increasing levels of Exch-Co from west to north-east (Fig. 3a). The category 0.50 to 0.67 mg/kg covers nearly half of the area with 0.67 to 0.84 mg/kg closely following it. The Carb-Co increases, in general, from south-west to north-east and towards south-east (Fig. 3b). The maximum area is covered by the 0.074 to 0.122 mg/kg level closely followed by 0.122 to 0.170 mg/kg.

Fig 3. Isorithmic maps of exchangeable cobalt (mg/kg) (a) and carbonate cobalt (mg/kg) (b).

Comparison of sample statistics

The additional values of Exch-Co and Carbo-Co were generated at unrecorded sites by point and block kriging, using semivariograms of these nutrients. A block size of 24 km x 24 km was used for interpolation because it gave the minimum estimation variance compared to other block sizes. The estimates of means by kriging (point & block) and classical technique were also same for both the forms. However, point kriging estimation variances were 1.3 to 1.5 times less than the sample variance (Table 1). The estimated variances obtained by block kriging were 12.5 to 15 times less than the sample variances, demonstrating an improvement in estimation precision by kriging methods over the classical methods. It could therefore be inferred that kriging could explain most of the variation in the original data. Practically it means that if a variable is spatially dependent, the estimation of variance by the classical method is not a reliable parameter for the interpretation of the data. Further, the ratio of the point and block kriged variances exhibited ten fold difference for these nutrients (Table 1).
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Table 1. Comparison of statistical parameters of exch-Co and Carb-Co using different approaches

<table>
<thead>
<tr>
<th>Statistical parameter</th>
<th>Exch-Co</th>
<th>Classical technique</th>
<th>Carbo-Co</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (m)</td>
<td>0.63</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Variance (s²)</td>
<td>0.05</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>Standard deviation (s)</td>
<td>0.23</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>Mean (mk)</td>
<td>0.63</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Variance (s²k)</td>
<td>0.04</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>Standard deviation (sk)</td>
<td>0.20</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Mean (mkb)</td>
<td>0.63</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>Variance (s²kb)</td>
<td>0.04</td>
<td>0.0004</td>
<td></td>
</tr>
<tr>
<td>Standard deviation (skb)</td>
<td>0.07</td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>

| s²/s² | 1.30 | 1.50 |
| s²/s²k | 12.50 | 15.00 |
| s²/s²kb | 10.00 | 10.00 |

By comparing means, variances and Student's t-values, we validated kriging for these spatially dependent cobalt forms. Validation could also be done from a mean estimation error (e'), a variance of errors (s²e) and from standard variation of errors (s_e). The kriged values (point & block) were tested and validation was found to be better for block kriged estimates because of their lower standard errors (Table 2).

Table 2. Statistical parameters for testing the zero mean estimation error (unbiased) and low variance conditions for the kriging estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>e'</th>
<th>s²_e</th>
<th>s</th>
<th>t</th>
<th>e'+2S_e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exch-Co</td>
<td>0.0012</td>
<td>0.042</td>
<td>0.21</td>
<td>0.048</td>
<td>95.8</td>
</tr>
<tr>
<td>Carb-Co</td>
<td>-0.0003</td>
<td>0.006</td>
<td>0.08</td>
<td>0.29</td>
<td>95.8</td>
</tr>
</tbody>
</table>

t_{0.95} = 2.093 e' = mean estimation error; s²_e = variance of errors; s_e = standard deviation of errors

Designing an optimal sampling scheme

Geostatistically analysed data can be used for designing an optimal sampling scheme for a given parameter (Webster 1985). The standard deviations of observed and kriged estimates were used for calculating the number of samples required to improve precision within +10% of the true mean at 90, 95 and 99% confidence levels. Block kriging reduced the sample size 12 times than classical and 9 times than point kriging methods (Fig. 4a,b)
indicating that only 3 (Exch-Co) and 7 (Carb-Co) samples would be sufficient to obtain the same precision at 90% confidence level. This could considerably reduce sampling efforts and cost.

Fig. 4. Diagrams showing the advantage of block kriging.

References


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